

The Hardness of the Lemmings Game, or Oh no, more NP-Completeness Proofs

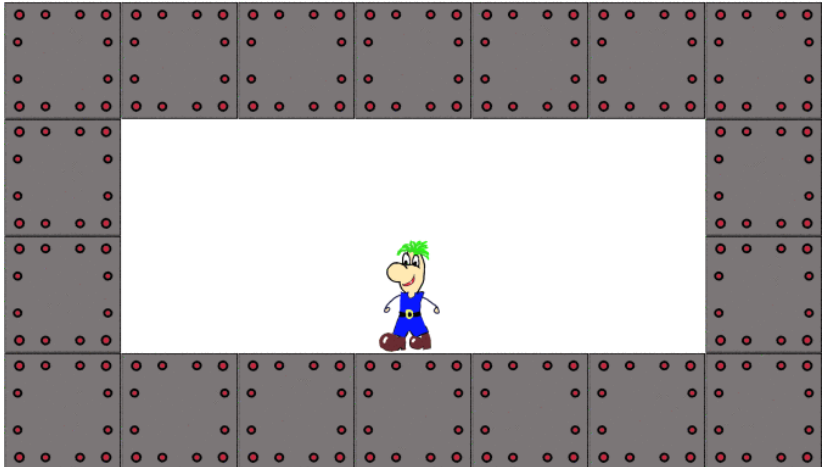
Dominika Salawa

June 13, 2019

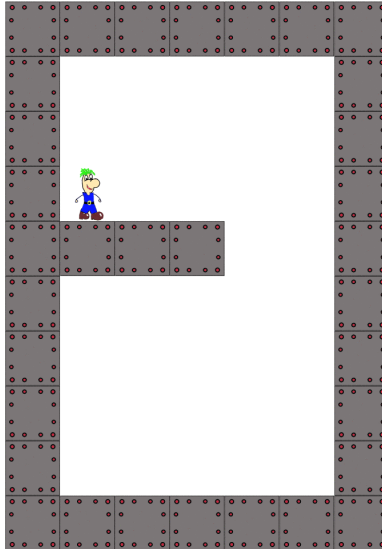
Lemmings

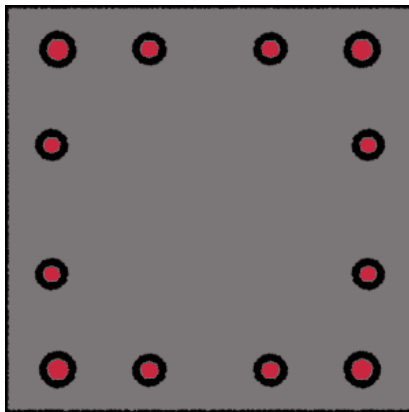


Walking

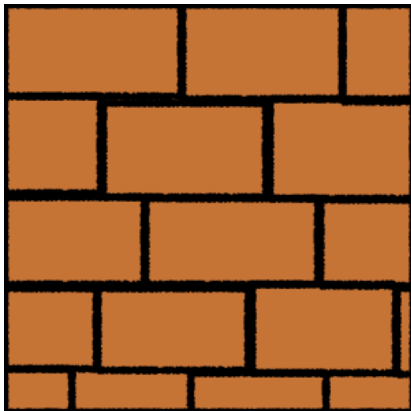


Falling

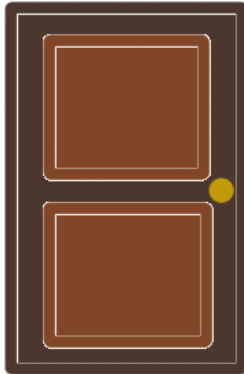


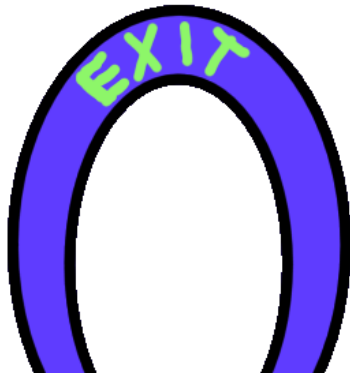


Permeable



Entrance





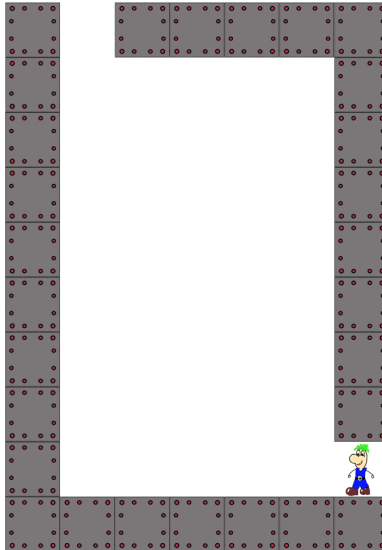
Skills:

- Permanent
- Semi-permanent
- Temporary

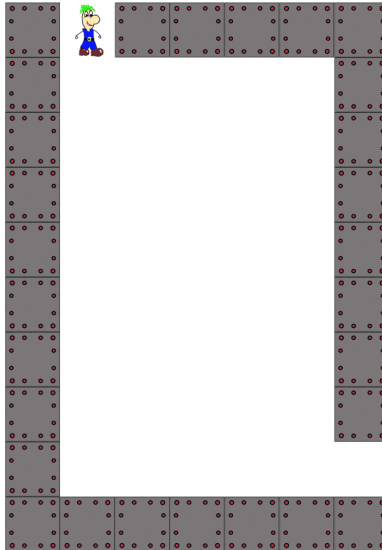
Permanent skills:

- Climber
- Floater

Climber



Floater

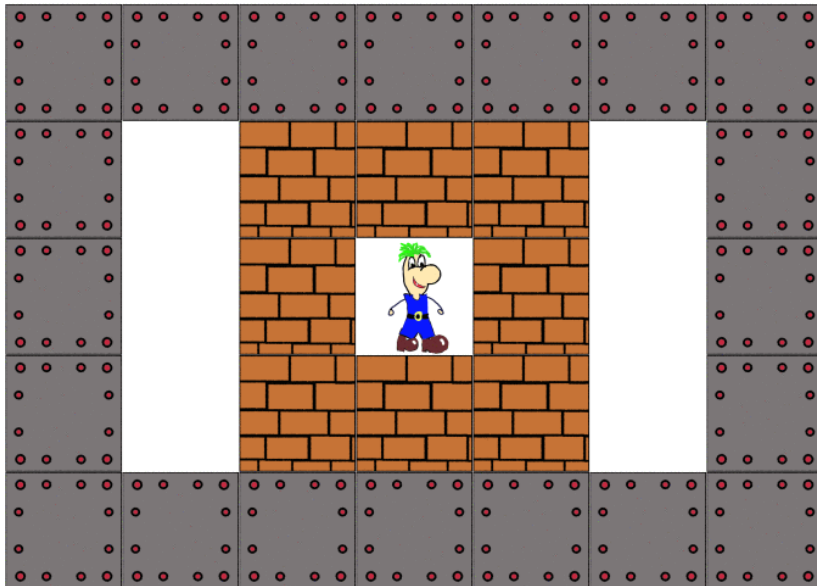


Semi-permanent skills

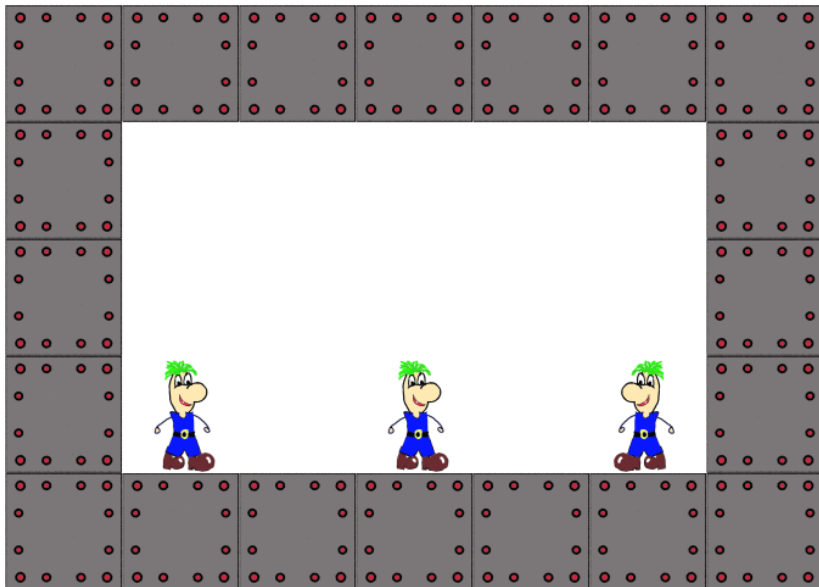
Semi-permanent skills:

- Exploder
- Blocker

Exploder



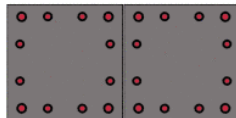
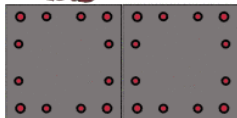
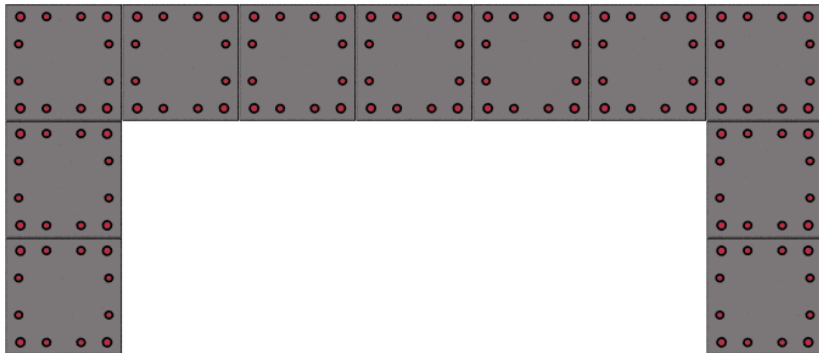
Blocker



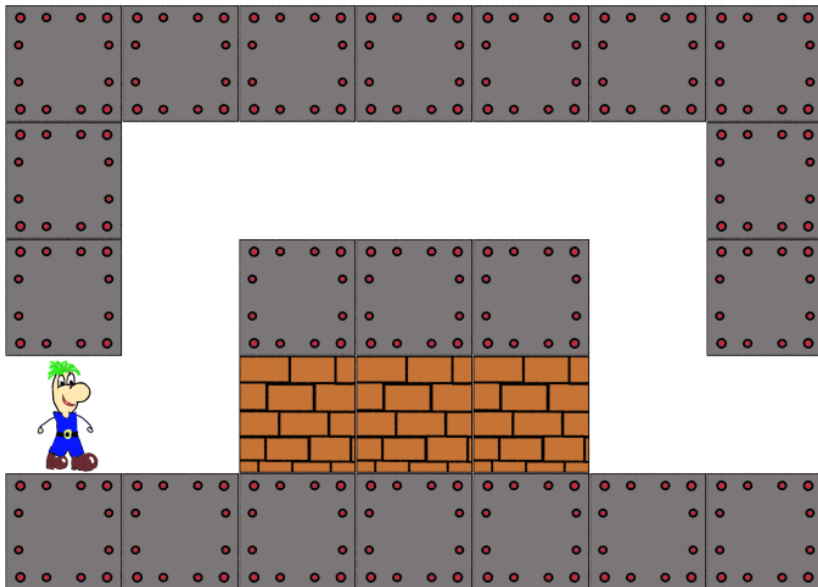
Temporary skills:

- Builder
- Basher
- Digger
- Miner

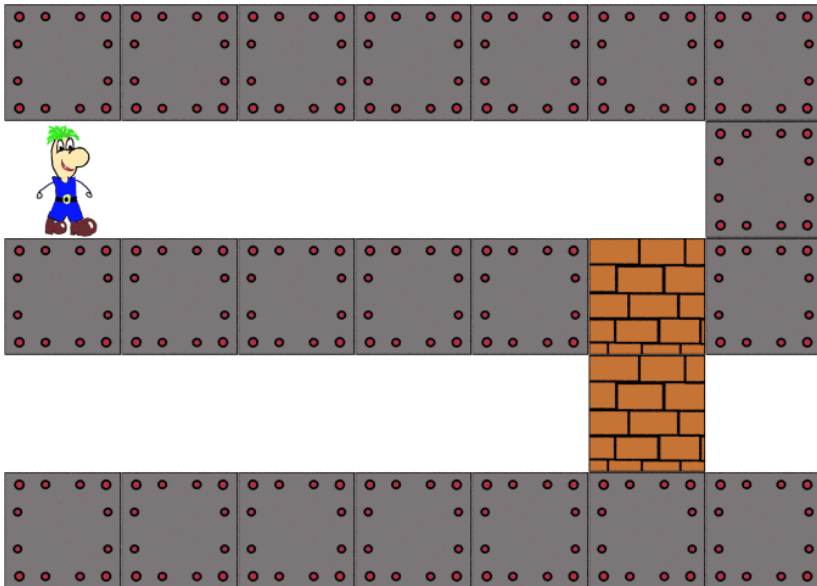
Builder

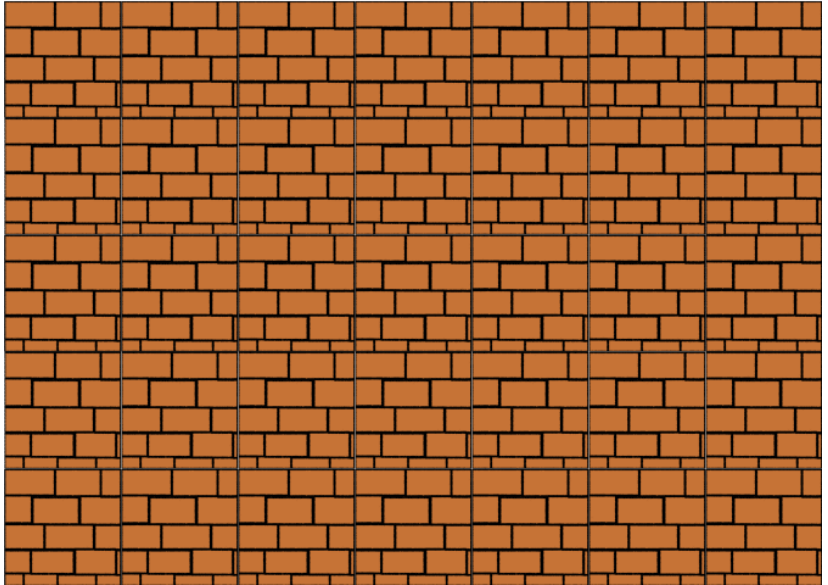


Basher



Digger





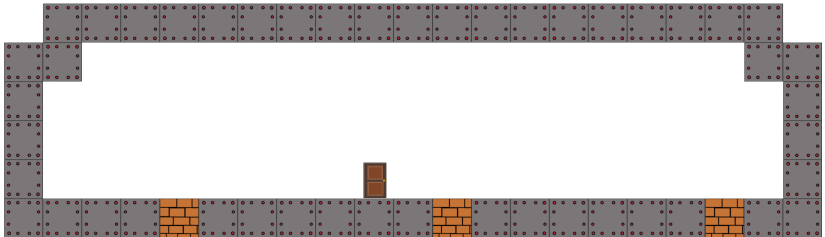
The Decision Problem

$L = (\text{limit, save, lems, start, width, height, grid, exit, skills})$

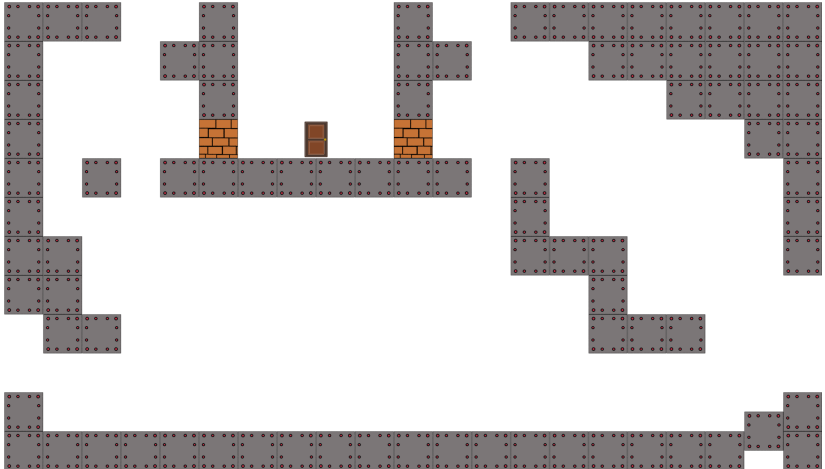
LEMMINGS \in NP
?

LEMMINGS \in NP-HARD

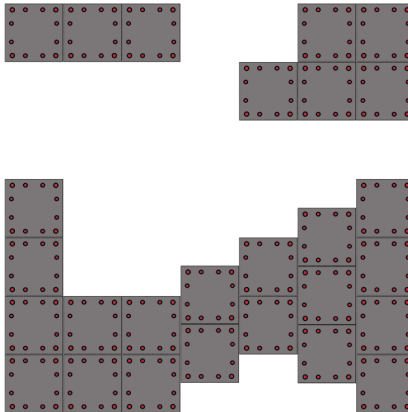
Gadgets: Clause



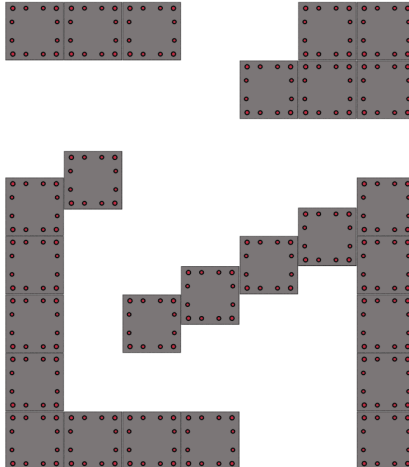
Gadgets: Variable



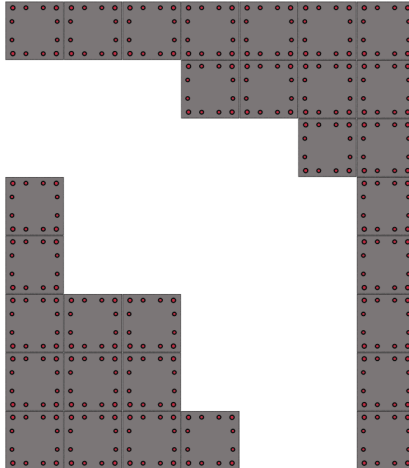
Gadgets: Junction



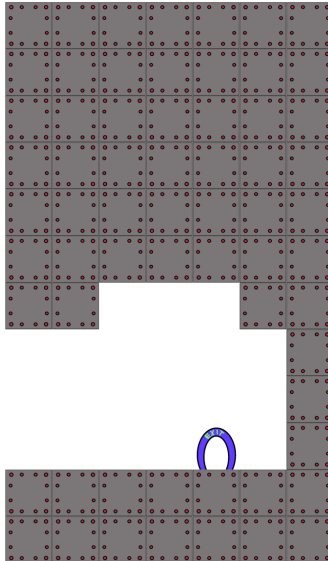
Gadgets: Wire



Gadgets: Corner



Gadgets: Exit



$$(\neg v_2 \vee v_3 \vee v_1) \wedge (v_1 \vee \neg v_2 \vee \neg v_3) \wedge (\neg v_1 \vee \neg v_3 \vee \neg v_2)$$

Construction



$$(\neg v_2 \vee v_3 \vee v_1) \wedge (v_1 \vee \neg v_2 \vee \neg v_3) \wedge (\neg v_1 \vee \neg v_3 \vee \neg v_2)$$

Construction



v_1

$\neg v_1$

v_2

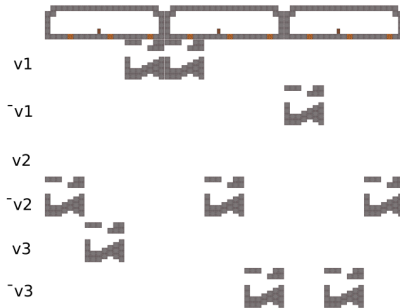
$\neg v_2$

v_3

$\neg v_3$

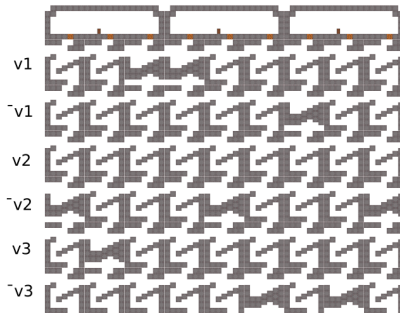
$$(\neg v_2 \vee v_3 \vee v_1) \wedge (v_1 \vee \neg v_2 \vee \neg v_3) \wedge (\neg v_1 \vee \neg v_3 \vee \neg v_2)$$

Construction



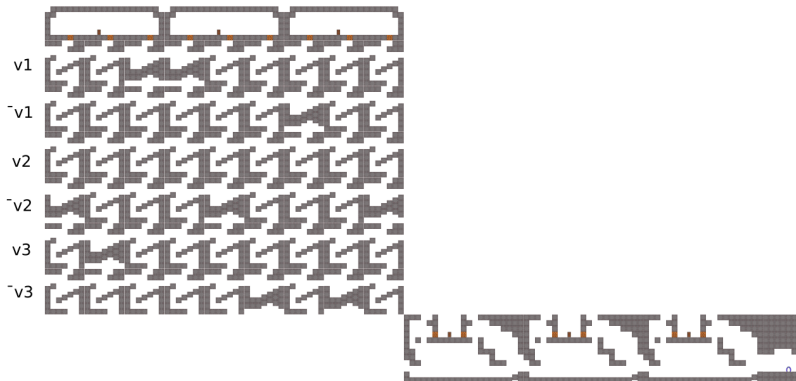
$$(\bar{v}_2 \vee v_3 \vee v_1) \wedge (v_1 \vee \bar{v}_2 \vee \bar{v}_3) \wedge (\bar{v}_1 \vee \bar{v}_3 \vee \bar{v}_2)$$

Construction



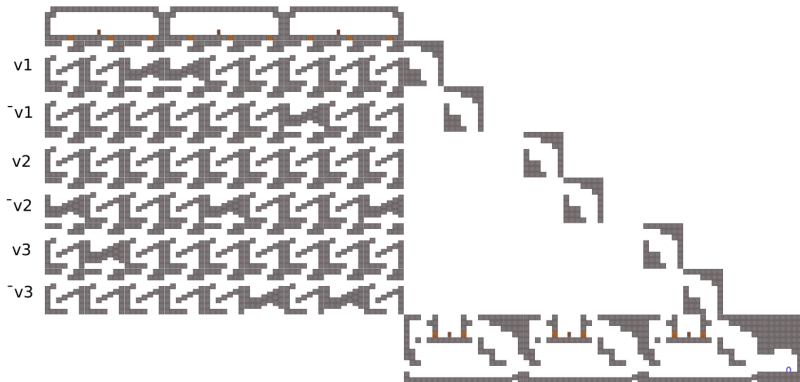
$$(\sim v2 \vee v3 \vee v1) \wedge (v1 \vee \sim v2 \vee \sim v3) \wedge (\sim v1 \vee \sim v3 \vee \sim v2)$$

Construction



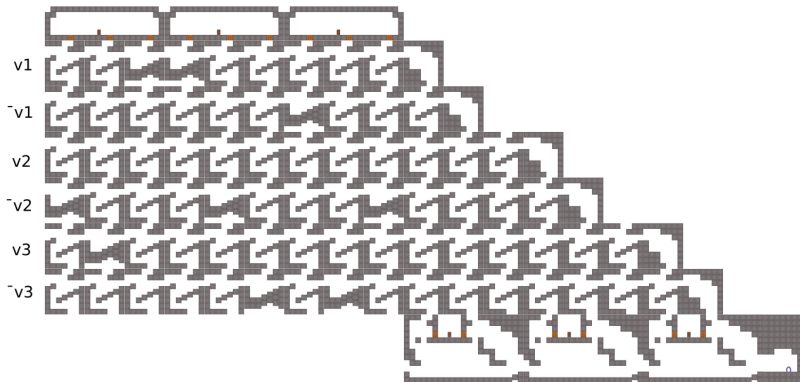
$$(\neg v_2 \vee v_3 \vee v_1) \wedge (v_1 \vee \neg v_2 \vee \neg v_3) \wedge (\neg v_1 \vee \neg v_3 \vee \neg v_2)$$

Construction



$$(\neg v_2 \vee v_3 \vee v_1) \wedge (v_1 \vee \neg v_2 \vee \neg v_3) \wedge (\neg v_1 \vee \neg v_3 \vee \neg v_2)$$

Construction



$$(\neg v_2 \vee v_3 \vee v_1) \wedge (v_1 \vee \neg v_2 \vee \neg v_3) \wedge (\neg v_1 \vee \neg v_3 \vee \neg v_2)$$